Violating the Standard Assumption Underlying the Process Monitoring: Perfect Measurements

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ABSTRACT

The presence of the measurement errors is a rule rather than an exception in any process monitoring context. Since the observations are generally observed, in a quick manner, by some measuring process and are time ordered. Moreover, their presence contributes negatively towards the performance of the usual control charting schemes. Therefore, it will be naïve and may lead to invalid conclusions to devise a control structure, without taking them into account, at first place, and then counteracting their adverse effect. This paper studies the foregoing effect on the performance of two eminent quality control charts (QCC), for the mean or average parameter of a process, namely: Crosier CUSUM and Dual CUSUM charts. With the growing intensity of the error variation, a deteriorating performance of the said charts has been observed. The average run length (ARL) behaviors, zero-state and steady-state of both charts are studied, using the Monte-Carlo simulation algorithm, under different situations for the effect of measurement errors. Also, extra quadratic loss (EQL), relative average run length (RARL), and performance comparison index (PCI) are used to evaluate the performance of the charts. Repeated measurements are employed, as a remedial scheme, for the effect of measurement errors. An illustrative example is incorporated to elucidate the study.

Keywords: CUSUM, Crosier CUSUM, Dual CUSUM Control Charts, Error Variation, Monte-Carlo Simulations, Performance Measure, Process Monitoring, Measurement Errors.

1. INTRODUCTION:

Statistically speaking, there are two types of variations that may violate the smooth running of a manufacturing or servicing process: random or natural and causal variations. The former are the organic and uncontrollable part of the process, and a process is called in-control (IC) if it is working under such variations only. The causal variations are those which need to be immediately detected and accounted for. If such variations are at work, the process is called out of control (OC).

Quality control charts or simply control charts (CC) are the most widely used and effective devices, in the so called statistical process control tool kit. They are meant to monitor as well as improve the quality of a product or a service. They do so by signaling timely any departure from normalcy, due to the presence of some assignable causes in a process. Such departure or aberration is generally known as shift (in process parameters) in quality control literature. The idea of control chart was first conceived by Shewhart (1920), to detect shifts in the mean of a manufacturing process. A CC comprises a target value for the process parameter along with a tolerable range, for that value, known as control limits. Samples are repeatedly drawn from the process and values for the concerned parameter are estimated. If the values are within the control limits, the process is considered as IC, otherwise OC. Since its intervention in the industrial and servicing processes, many modifications and improvements of CC, have been suggested. Mainly, there are two types of control charts: memory-less and memory-type control charts. As the names suggest, the former is based upon only current sample, therefore, suitable for large shift size (more than 1.5 times standard deviation (SD) of the process); whereas the latter takes into account the past samples as well, and is therefore suitable for small sized shifts (Montgomery, 2020). Shewhart CC and its different variants are examples of memory-less charts. The memory-type control charts mainly constitute of cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts. The latter, devised by (Roberts, 1959), is based upon the weighted average of current and all the past observations, with geometrically assigned weights, such that the current value has the highest one. See also (Crowder, 1987), (Crowder, 1989) and (Lucas & Saccucci, 1990) for good discussion of the EWMA. An efficient CC should be sensitive enough to signal any shift (small or large) as soon as possible and should be robust enough for the process settings.

First introduced by (Page, 1954), the CUSUM chart, based on the cumulative sum of deviations from the mean, is used to detect small to moderate shifts, in a process. Two statistics work simultaneously for both depreciation and rise in the target value of the concerned parameter. Many a research article have been produced to explore and improve different aspects of the CUSUM chart. (Lucas, 1982) combined the Shewhart and the CUSUM to construct a single chart that was efficient to signal both large and small shifts, in a process. (Lucas & Crosier, 2000) introduced the idea of fast initial response (FIR), by using a headstart, which made it more sensitive at the start. (Jiang et al., 2008) originated the idea of adaptive CUSUM, which first estimated the expected shift size in the process and accordingly constructed a plausible CUSUM CC. Similarly (Riaz, 2008) for the first time made use of the auxiliary information (variable) for the estimation of the quality parameter, taking inspiration from survey sampling techniques. (Sales et al., 2020) proposed a Shewhart control chart for monitoring the mean under a first order Poisson mixed autoregressive process.

(Crosier, 1986) skillfully amended, the structure of the CUSUM to make it based on a single statistic. This not only simplified the multivariate version of the CUSUM but also showed a little improvement in the performance than the classical one. (Zhao et al., 2005) presented the idea of the dual CUSUM

(DCUSUM), by maneuvering the (Lorden, 1971) strategy of running infinite CUSUM charts for various shift sizes. The rationale of DCUSUM being that the CUSUM chart is optimal for only a given shift size, in the process. He argued that running two, instead of one, CUSUM charts would enhance the sensitivity of the controlling scheme, to detect optimally a number of shifts lying within a specified range.

In any measuring situation, various ways, some known some yet-to-be-known, are available to record the characteristic of interest. Each of these measurements, results into a similar but not necessarily the same observation. This disagreement between the true and the observed value of the concerned characteristic is referred to as measurement error. (Bennett, 1954), for the first time studied the effect of these errors on the process monitoring and concluded that the measurement variance that was less than the variance of the process was negligible. He used the simplest model for the errors, which is the difference between the true and the measured values of the quality variable. Following the same model, (Abraham, 1977) elaborated the effect of these errors on the control limits of the mean chart; (Kanazuka, 1986) saw the same effect on combined Mean-Range chart, in terms of power structure while (H-J Mittag, 1995) and (Hans-Joachim Mittag & Stemann, 1998) on the S-chart, instead of R-chart. (Linna & Woodall, 2001) used a covariate model for the effect of measurement errors on the performance of mean and variance charts. (Stemann & Weihs, 2001) investigated the effect on the EWMA chart. (P. Maravelakis et al., 2004) considered the covariate model for the effect of these errors on the EWMA chart; and (P. E. Maravelakis, 2012) studied this effect on the CUSUM chart and observed that the CUSUM performed better than EWMA in the presence of measurement errors. For more discussion, we refer to: (K. P. Tran et al., 2016), (Sabahno & Amiri, 2017), (P. H. Tran & Heuchenne, 2019), (Sabahno et al., 2019), (Saha et al., 2020), (Zaidi et al., 2020), (Nguyen et al., 2020), (Arif et al., 2020), and (Ayyoub et al., 2020).

Both Crosier's CUSUM and dual CUSUM charts are more powerful than the conventional one, in detecting shifts in the location parameter of a process. Of late, extensive research includes exploring the different features of these two charts and amalgamating new ideas with these to have more powerful tools for the statistical process control. This paper seeks the performance of CCUSUM and DCUSUM under the effect of the measurement errors. The performance is evaluated in terms of average run length (ARL), standard deviation of run length (SDRL) and median run length (MRL), using Monte-Carlo simulation algorithm. Both zero-state and steady state behaviors are studied. Furthermore, extra quadratic loss (EQL), relative average run length (RARL) and performance comparison index (PCI) are also used to evaluate the performance of the charts. The rest of the paper proceeds as: Section 2 discusses briefly the structure of the usual CCUSUM and DCUSUM and the models for effect of measurement errors; Section 3 sees the effect of measurement errors on the two charts; Section 4 includes the simulation study of the effect; Section 5 compares the performance of the two charts under perfect and faulty measurements; finally, Section 6 contains concluding remarks and recommendations.

2 Existing Charts

Let $\{ X_t;\ t \geq 1\}$ be the underlying quality characteristic, which is $N(\mu,\ \sigma^2)$ at time, $t \geq 1$. In order to monitor the changes in the process mean μ, a simple random sample of size n: (X_{1t}, X_{2t}, … , X_{nt}), is repeatedly drawn, from the process, and the value of $\bar{X}_t = \frac{1}{n}\sum_{i=1}^{n} X_{it}$, the sample mean at the time t, is computed. In correspondence to the $\{X_t\}$, $\{\bar{X}_t;\ t \geq 1\}$ is a sequence of independently and identically normally distributed random variables with mean μ and variance $\frac{\sigma^2}{n}$. Suppose that the process $\{X_t\}$, remains in IC state till a particular point of time and then gets out-of-control because of the occurrence of an unknown shift δ, in the process mean μ. Here δ is the standardized shift: $\delta = |\mu_1 - \mu_0|/(\sigma\, n^{-0.5})$, corresponding to the new out-of-control mean of the process 'μ_1'.

2.1 Crosier's CUSUM chart under the assumption of perfect measurements

The CCUSUM chart is a modification of the conventional CUSUM, which first updates the cumulative sum of deviations from mean and then shrinks this sum to zero, by means of multiplication/division rather than addition/subtraction. Based upon a single statistic, $\{A_t\}$, the structure of the CCUSUM is as follows:

$$\left.\begin{array}{ll} A_t = 0 & if \quad C_t \leq K \\ A_t = (\bar{X}_t - \mu + A_{t-1})(1 - K/C_t) & if \quad C_t > K \end{array}\right\} \tag{1}$$

where $C_t = |A_{t-1} + (\bar{X}_t - \mu)|$ is the updated sum with $A_0 = 0$, and K is the usual reference parameter. A one sided CCUSUM signals, no sooner than $A_t > H$, where H is the control limit of the CCUSUM.

2.2 Dual CUSUM chart under the assumptions of perfect measurements

A handicap of the CUSUM chart and its variants is that they are only optimal for a given shift size, to be known or assumed in advance. (Zhao et al., 2005) argued, since the size of the shift couldn't be known for sure before its occurrence, therefore, it would be more appropriate to assume that the shift might occur uniformly within a specified range, say, $[a, b]$. Under this assumption, he proposed of running two, instead of one CUSUM, for better capability of the control scheme, to detect changes in the parameter. A one sided DCUSUM, based on $\{\bar{X}_t\}$, includes two charts $\{D_{1,t}\}$ and $\{D_{2,t}\}$, running simultaneously:

$$\left.\begin{array}{l} D_{1,t} = \max[0, D_{1,t-1} + (\bar{X}_t - \mu) - K_1] \\ D_{2,t} = \max[0, D_{2,t-1} + (\bar{X}_t - \mu) - K_2] \end{array}\right\} \tag{3}$$

where $D_{i,0} = 0$, $K_i = k_i\sigma/\sqrt{n}$ (for $i = 1, 2$) is the reference value. The one-sided DCUSUM sets off on out-of-control signal when $D_{i,t} > H_i$ (for $i = 1, 2$), where $H_i = h_i\sigma/\sqrt{n}$ is the control limit of the DCUSUM chart. In order to work efficiently, DCUSUM scheme requires the following assumptions met:

(i) The shift size δ lies uniformly within [a, b].

(ii) $k_1h_1 = k_2h_2$ and $k_1 + h_1 > k_2 + h_2$

(iii) $k_1 = (3a + b)/8$ and $k_2 = (a + 3b)/8$.

For more details vide (Zhao et al., 2005).

3 Effect of measurement errors on the performance of control charts

The discrepancy between the observed or recorded value and the actual or true value of a characteristic is called measurement error. This error creeps into a study by a number of reasons e.g. inability of the measuring instrument, human or machine error, proxies or surrogates, etc. Once occurred, through whatever source, these errors cannot be detected and measured. Therefore, one has to be vigilant of and accounted for these errors. The effect of the measurement errors is manifold: non-representative average, lack of precision, inflated variance and consequent false conclusions, etc. A frequent way of countering these errors is to use multiple observations per sampling unit. But for cost consideration, the number of measurements per unit is restricted to five. A number of publications are available on the effect of measurement errors on the performance of the control charts. (Linna & Woodall, 2001) comprehensively studied the effect of the errors on the performance of the Shewhart charts. They considered the simple linear model, with and without a covariate, to evaluate the effect of these errors. They concluded that the standardized shift in the covariate parameter was always smaller than that in the process parameter. Moreover, the power of the chart could be raised by using multiple measurements per unit. (P. E. Maravelakis, 2012) observed the error effect on the classical CUSUM chart and concluded that under the effect of measurement errors, the CUSUM chart performed better than EWMA chart to detect smaller shifts. The most common situations of the errors is represented by a simple linear model:

$$Y_{it} = X_{it} + e_{it}, \quad (1)$$

where X_{it} is the actual/true value of the underlying quality characteristic, as defined above; Y_{it} is the observed /measured value of the quality characteristic; and e_{it} is the measurement error, which is uncorrelated with X_{it}, and is identically and independently $N(0,\ \sigma_m^2)$. (Linna & Woodall, 2001) used a covariate for the measurement of the X_{it}:

$$Y_{it} = A + BX_{it} + e_{it.}$$

The former is a special case of the latter with $A = 0$ and $B = 1$.

Following the model given in eq. (1), it is obvious that $Y_{it} \sim N(0, \sigma^2 + \sigma_m^2)$. The sample mean $\bar{Y}_t$ which is the concerned statistic, becomes;

$$\bar{Y}_t = \frac{\sum_{i=1}^{n} Y_{it}}{n} = \frac{\sum_{i=1}^{n}(X_{it} + e_{it})}{n}$$

$$\bar{Y}_t = \bar{X}_t + \bar{e}_t \text{ and}$$

$$\sigma^2(\bar{Y}_t) = \frac{1}{n}(\sigma^2 + \sigma_m^2) \quad \text{or}$$

$$= \frac{\sigma^2}{n}(1+\gamma), \text{ where } \gamma = \frac{\sigma_m^2}{\sigma^2}$$

$$\sigma(\bar{Y}_t) = \frac{\sigma}{\sqrt{nc}}, \text{ where } c = \sqrt{\frac{1}{1+\gamma}}$$

$$\bar{Y}_t \sim N(\mu, \sigma^2/nc^2).$$

Let the process work under control for a certain time period and then a shift occurs in the mean of X_t as: $\mu_1 = \mu + \delta \frac{\sigma}{\sqrt{n}}$ or $= \left[\frac{\mu_1 - \mu}{\sigma}\right]\sqrt{n}$, which will in turn transform into $\bar{Y}_t$ as $\mu_1 = \mu + \delta \frac{\sigma}{\sqrt{nc}}$ or $\delta_y = \left[\frac{\mu_1 - \mu}{\sigma}\right]\sqrt{nc}$ or $\delta_y = \delta c$. Since the value of c remains between zero and one (being one for no measurement error and is inversely proportion to γ), the shift in $\bar{X}_t$, is always more than that in $\bar{Y}_t$, whenever there is measurement error. It follows that the presence of measurement error will reduce the size of the shift occurred and consequently the sensitivity of the control chart.

As mentioned before, the effect of measurement errors can be reduced by using a number of measurements per unit of the quality characteristic. For repeated measurements, the model becomes:

$$Y_{ijt} = X_{it} + e_{ijt},$$

where $j = 1,2, \dots, m$, shows the number of measurements per unit. The sample mean and its standard error become:

$$\bar{Y}_t = \frac{\sum_{i=1}^{n}\sum_{j}^{m} Y_{ijt}}{mn} = \frac{\sum_{i=1}^{n}\sum_{j}^{m}(X_{it} + e_{ijt})}{mn}$$

$$\bar{Y}_t = \frac{1}{mn}\left(m\sum_{i=1}^{n} X_{it} + \sum_{i=1}^{n}\sum_{j}^{m} e_{ijt}\right)$$

$$\bar{Y}_t = \bar{X}_t + \bar{e}_t$$

$$\sigma_{\bar{Y}_t} = \sqrt{\frac{\sigma^2}{n} + \frac{\sigma_m^2}{mn}}$$

$$= \sqrt{\frac{1}{n}\left(\sigma^2 + \frac{\sigma_m^2}{m}\right)}$$

$$= \sqrt{\frac{\sigma^2}{n}\left(\frac{m+\gamma}{m}\right)}$$

$$= \frac{\sigma}{\sqrt{nc}} \text{ where } c = \sqrt{\frac{m}{m+\gamma}}$$

Clearly, c increases as m increases, therefore, more the measurements per unit, less the $S.E(\bar{Y}_t)$.

3.1 Effect of measurement errors on CCUSUM and DCUSUM Chart

The foregoing discussion makes it quite obvious that the performance of a control chart, whether memory type or memory-less, is adversely affected by the presence of measurement variation. The two titled charts are no exception to that. This section and next will see the same effect on the performance of the two charts namely: Crosier's CUSUM and Dual CUSUM. The general format of the charts is, more or less, same. The only difference being that the observations Y_t, unlike X_t, are infected with the measurement errors. At first place, for simplicity, the basic model given in eq. (1) is used. The slightly changed format of CCUSUM is as follows:

$$\left.\begin{array}{ll} A_t = 0 & if \quad C_t \le K \\ A_t = (\bar{Y}_t - \mu + A_{t-1})(1 - K/C_t) & if \quad C_t > K \end{array}\right\} \quad (4)$$

and the DCUSUM becomes:

$$\left.\begin{array}{l} D_{1,t} = \max[0, D_{1,t-1} + (\bar{Y}_t - \mu) - K_1] \\ D_{2,t} = \max[0, D_{2,t-1} + (\bar{Y}_t - \mu) - K_2] \end{array}\right\} \quad (5)$$

where $\bar{Y}_t$ is as defined previously in eq (4).

4 Performance measures and analysis

In control charting studies, the performance evaluation of the proposal has always been a crucial and significant part of the research. Here, the evaluation process is carried out using different performance measures, derived from different properties of the run length distribution of the controlling scheme, namely: average run length (ARL), standard deviation of RL (SDRL), median of RL (MDRL), extra quadratic loss (EQL), relative ARL (RARL) and performance comparison index (PCI).

Average Run Length (ARL) : A control chart is usually evaluated in terms of the average of the run length distribution (ARL). A run is the number of consecutive samples, for which the process remains in control. The ARL is usually defined as the reciprocal of the probability that the value of the statistic, used for process monitoring, falls outside the control limits. It refers to as the number of samples before a shift is observed, when the process is either in control or out of control. Generally, it is assumed that the control scheme starts at its initial value and the process is in control, for the computation of ARL. Such an ARL is called zero-state ARL. A steady-state ARL (SSARL), on the other hand, is computed by assuming that the process has been working for quite some time, when the scheme is applied, which implies that the initial value of the scheme may not be zero. More information can be found in (James M Lucas & Crosier, 2000) and (James M Lucas & Saccucci, 1990). The ARL for a chart should be as high as possible, when the process is in control; and should be as low as possible, when the process is out of control. The ARL is used as a comparative measure, among different control charts, for a particular shift size. It is computed either by using Markov Chain approach or by Monte Carlo simulation. The current paper uses the latter.

Extra Quadratic Loss (EQL): The extra quadratic loss (EQL) is an alternative performance measure that appraises the overall performance of a control chart, against the whole domain of shifts. The EQL is defined as a weighted average ARL over the whole process shift domain $[\delta_{min}, \delta_{max}]$ using the square of shift (δ^2) as a weight. The minimum EQL value of a chart construes that the chart is the best among all. It is assumed that the probability distribution of δ is uniform with density function $(\delta_{max} - \delta_{min})^{-1}$ over the entire shift range $[\delta_{min}, \delta_{max}]$. Mathematically the EQL is given as:

$$\mathrm{EQL} = (\delta_{max} - \delta_{min})^{-1} \int_{\delta_{min}}^{\delta_{max}} \delta^2 \mathrm{ARL}(\delta) d\delta, \quad \forall\, \delta \in [\delta_{min}, \delta_{max}]$$

Relative Average Run Length (RARL) : For the comparison of several control charts over a range of shift sizes, (Zhao et al., 2005) introduced another measure, known as IRARL or simply RARL (integrated relative ARL). Mathematically it is computed as:

$$\mathrm{RARL} = E\left[\frac{ARL_c(\delta)}{ARL_{opt.}(\delta)}\right] = (\delta_{max} - \delta_{min})^{-1} \int_a^b \mathrm{ARL}(\delta)/\mathrm{ARL}_{\mathrm{opt}}(\delta)\, d\delta$$

where $\delta \sim U(a, b)$, the numerator encompasses the ARL of the control chart and denominator covers the entire optimal ARL's, for the given range of shifts. Ideally, RARL must be one, but the control chart with the smallest value of RARL will be the best among the choices compared.

Performance Comparison Index *(*PCI*):* It is the ratio between the EQL of a chart and EQL of the best chart under the same conditions. This index facilitates the performance comparison by accomplishing ranking based on EQL. The chart with the lowest EQL has a PCI value equal to one, and the PCI values of all other charts are larger than one. Mathematically it can be written as $\mathrm{PCI} = EQL/EQL_{opt\ chart}$.

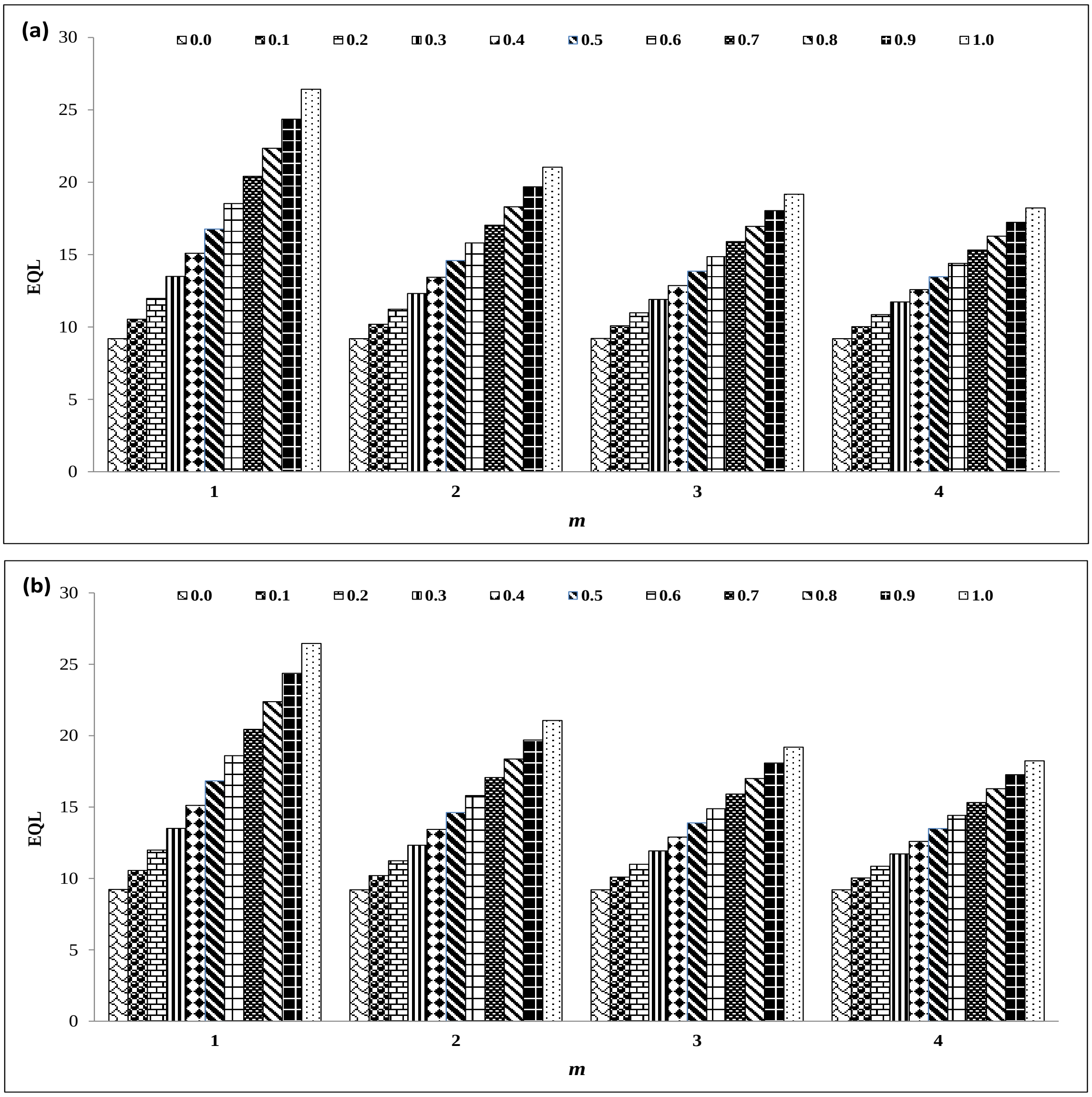
The above performance measures are also used by many authors in their studies including (Ahmad et al., 2014), (Abbas et al., 2016), (Zaman et al., 2020) and references therein.

4.1 Evaluation

The performance of the one-sided (upper), CUSUM, CCUSUM and DCUSUM charts is evaluated in terms of the ARL, in the presence as well as absence of measurement errors (the two sided versions can easily be extended on the same pattern). The in-control ARL is set at 370 and the ARL (both zero and steady states) for different shift sizes are computed, under different situations for measurement errors. With the support of the ARL values, other measures EQL, RARL and PCI are calculated to observe the overall performance of control charts.

The measurement error violation is studied in terms of ratio between error variance and the variance of the quality characteristic, namely γ. The ratio γ takes values between zero and one (both inclusive), with an interval of *0.1*. Zero means perfect measurements and 1 refers to the worst case of as large measurement variation as the variation in the quality characteristic itself. Different number of measurements (m = 1, 2, 3, 4) are also incorporated. The SSARL is based upon 32 samples before mean shifts. For DCUSUM, the values of k_1 and k_2 are respectively the lower and upper quartiles of the range $[a/2, b/2]$, that is, $k_1 = (3a + b)/8$ and $k_2 = (a + 3b)/8$.

The results are obtained by generating 100,000 random samples, from a standard normal distribution, of a given size. For each generated sample, the run length profile is calculated based on 100,000 replications. It is ensured that the results are invariant to the values of process parameters mean μ and standard deviation σ, and also to the sample size n. The out of control run length profiles ARL (ARL_1), SDRL and MDRL (at some specific shift sizes) of CUSUM, CCUSUM and DCUSUM charts (for zero state and (steady state)) are presented in the Tables: 1-4 (13-16), 5-8 (17-20) and 9-12 (21-24) respectively, at varying values of m. To illustrate further the performance of the three charts in the presence of measurement errors, the ARL curves are also drawn. These highlight the effect of the measurement errors on the ARL_1 of the charts, and the effect of the repeated measurements as a remedy for this effect. In order to examine the overall performance of the said charts, the EQL, RARL and PCI values are provided in the Tables 25 and 26 while the EQL bar charts are provided in Figures 4 and 5. The main findings are as under:



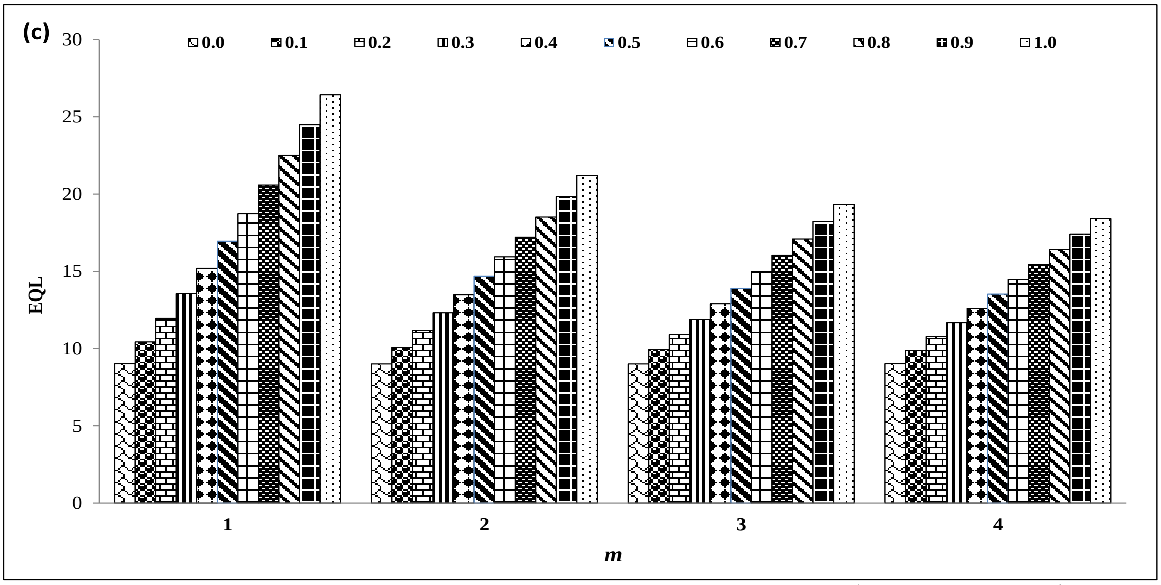


Figure 7: Zero-state EQL comparison for (a) CUSUM, (b) CCUSUM and (c) DCUSUM Charts at $m = 1, 2, 3$ and 4

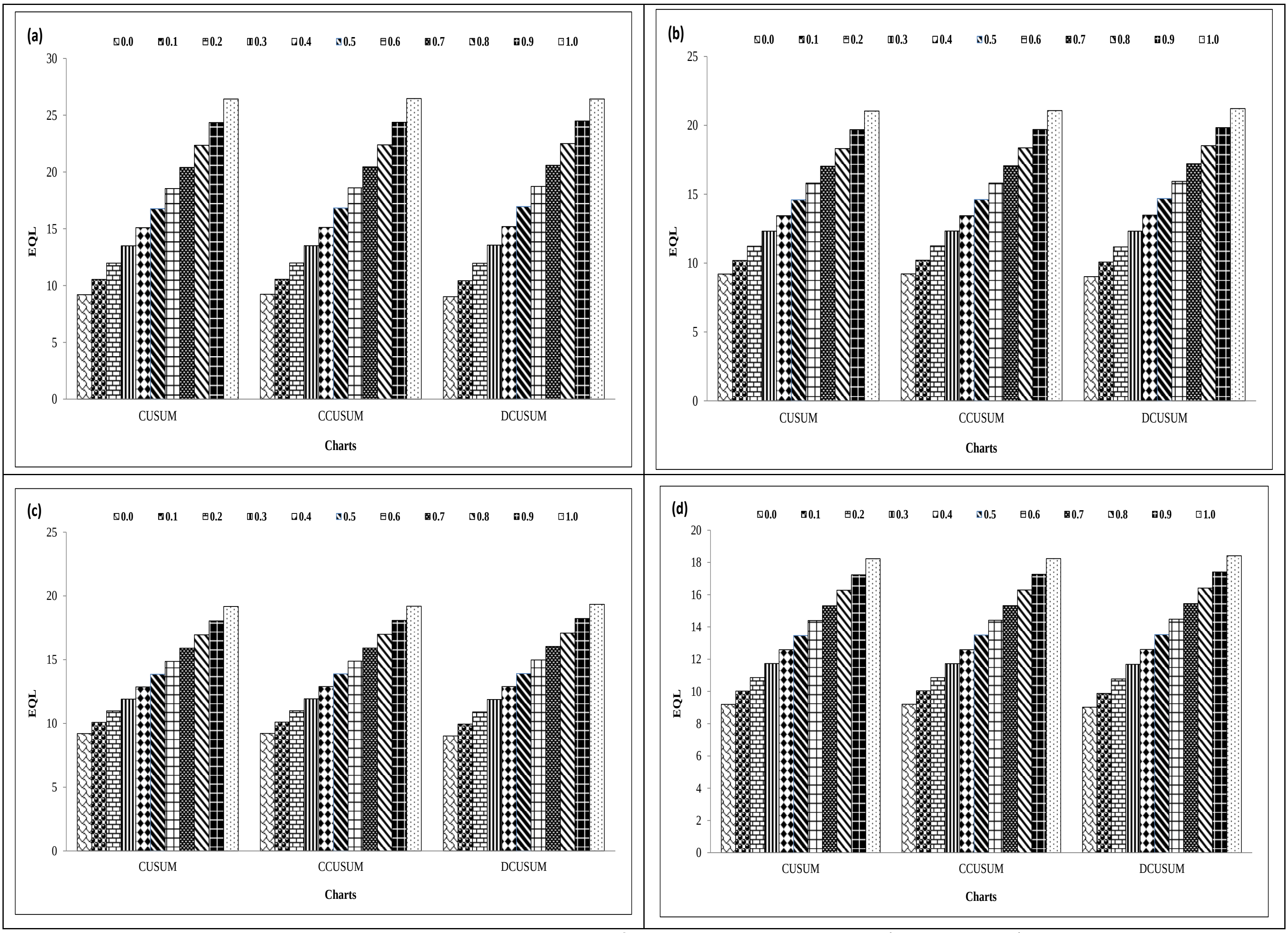


Figure 8: Zero-state EQL comparison for CUSUM, CCUSUM and DCUSUM Chart at (a) $m = 1$, (b) $m = 2$, (c) $m = 3$ and (d) $m = 4$

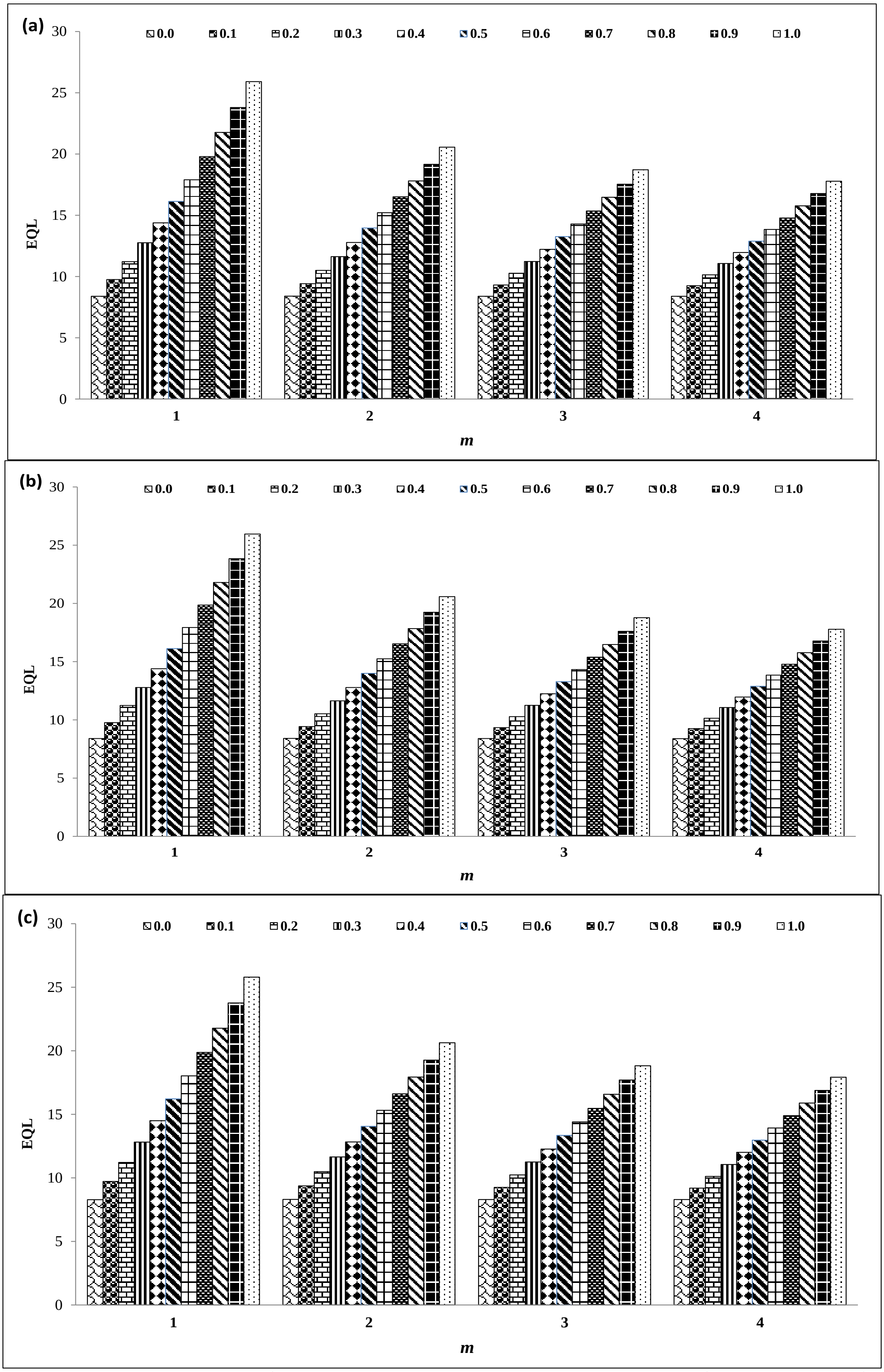


Figure 9: Steady-state EQL comparison for (a) CUSUM, (b) CCUSUM and (c) DCUSUM Charts at $m = 1, m = 2, m = 3$ and $m = 4$

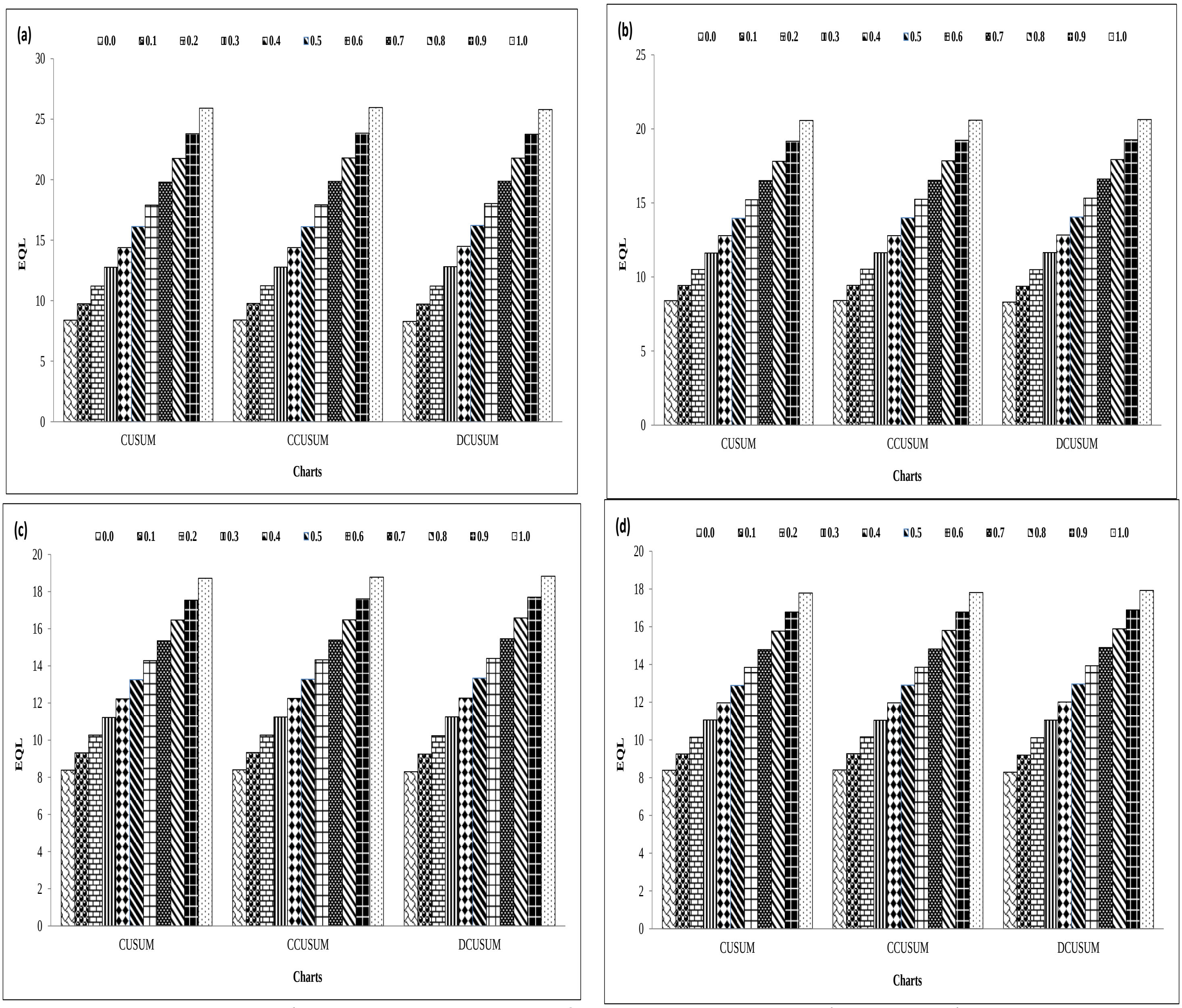


Figure 10: Steady-state EQL comparison for CUSUM, CCUSUM and DCUSUM Chart at (a) $m = 1$, (b) $m = 2$, (c) $m = 3$ and (d) $m = 4$

Shift $\delta \uparrow$ then ARL/SDRL/MDRL $\downarrow$: It can be observed from Tables 1-24 that the ARL values of CUSUM, CCUSUM and DCUSUM charts (at m=1, 2, 3 and 4) decrease as the values of shift increase from 0.2 to 2.0. As expected, having fixed in-control ARL, the out of control ARL/MDRL/SDRL is inversely proportional to the size of shift. For example we may observe (δ, ARL_1) from Table 1 as $(0.20,\ 106.91)$, $(1.00,\ 8.57)$ and so on $(2.00,\ 3.41)$. The similar decreasing behavior of SDRL and MDRL values can be observed in results; for example, from Table1, we can see (δ, SDRL) as $(0.20,$ $101.24)$, $(1.00,\ 4.77)$ and so on $(2.00,$ $1.17)$; and $(\delta,\ \mathrm{MDRL})$ as $(0.20,\ 76.00)$, $(1.00,\ 7.00)$ and so on $(2.00,\ 3.00)$. The similar behavior of ARL, SDRL and MDRL values (with respect to δ) can be observed in Table 2-24 at varying values of *m*. With the gradual increments of δ, exactly the same declining pattern of ARL_1 curves can be seen in the Figures: $1 - 6$, for $m = 1, 2, 3, 4$.

Ratio $\gamma \uparrow$ then ARL/SDRL/MDRL $\uparrow$: It can also be observed from Table 1-24 that the ARL values of CUSUM, CCUSUM and DCUSUM charts (at m=1, 2, 3 and 4) increase as the ratio γ increases from 0 (perfect case) to 1.0 (worst case). The out of control ARL (ARL_1) is directly proportional to the ratio γ, that is, more the measurement variation, higher the ARL_1. The SDRL and MDRL values also increase as the ratio γ increase. For example we may observe (γ, ARL_1) from Table 1 as $(0.00,\ 106.91)$, $(0.30,\ 140.81)$, $(0.60,\ 166.08)$ and so on $(1.00,\ 193.40)$. With the increment of $\gamma,$ the ARL_1 curves, all and sundry, shift upwards to the right. From Figures $1-6$, the left bottom curve is for $\gamma = 0$ and the right top most curve is for $\gamma = 1.0$.

The similar decreasing behavior of SDRL and MDRL values can be observed in the results: for example from Table1 we can see (γ, SDRL) as $(0.00,\ 101.24)$, $(0.30,\ 134.42)$, $(0.60,\ 161.32)$ and so on $(1.00,\ 187.65)$; and $(\gamma,\ \mathrm{MDRL})$ as $(0.00,\ 76.00)$, $(0.30,\ 99.00)$, $(0.60,\ 117.00)$ and so on $(1.00,\ 136.00)$. The similar behavior of ARL, SDRL and MDRL values (with respect to γ) can be observed in Table 2-24.

The simulation study conforms numerically to the established notion that repeated measurements lessen the measurement variation and thus decreases the ARL_1. That is, the values of the pair (δ, ARL_1) from Tables 1 $(m = 1)$ and Table 4 $(m = 4)$ are respectively: $(0.2, 157.91)$ and $(0.2, 140.07)$, for $\gamma = 0.5$. The phenomenon is endorsed by the figures too.

EQL, RARL, and PCI: The overall performance of CUSUM, CCUSUM and DCUSUM charts can be observed in Table 25 & 26 and Figures 7 & 8. The EQL values have same interpretation as ARL_1. For example, one may observe (γ, EQL), from Table 25 at $m = 1$ as, $(0.00,\ 9.202)$, $(0.30,\ 13.499)$, $(0.60,\ 18.536)$ and so on $(1.00,\ 26.422)$. It may also be observed that (at $\gamma = 0.10$, $m = 1, 2, 3, 4$) the respective EQL values for CUSUM chart are 10.547, 10.1954, 10.083 and 10.024; for CCUSUM chart, are 10.561, 10.207, 10.101 and 10.040; and for DCUSUM chart, are 10.435, 10.073, 9.947 and 9.881.

In Figure 7, the EQL values of each chart are directly proportion to γ as observed in point 2. The EQL values of each chart decrease as the value of *m* increases, which shows that the overall effectiveness of a chart is directly proportional to the number of measurements *m*. Moreover, the Figure 8 indicates that at a particular value of *m*, all charts have almost same overall detection ability.

Preference order, with respect to the sensitivity: A cursory overview of the Tables reveals the unaltered preference order, with respect to the sensitivity, among the three charts: DCUSUM is the most preferred one, followed by CCUSUM and CUSUM charts.

Contradictory to (Bennett, 1954) findings that the measurement variation less than the process variation is negligible, it suggests that measurement variation amounted to even 40% or 50% of the process variation, significantly aggravates the detecting power of a chart. For instance, to detect a shift of size one, CCUSUM and DCUSUM require respectively, 8.59 and 8.97 samples, when there is no measurement variation; while for the 50% measurement variation of the process variation, they require 16.71 and 16.93 samples respectively (cf. Tables: 5 and 9). A net percentage increase of 94.5 and 88.74 respectively.

5 An Illustrative Example (Application of Study)

In order to demonstrate the application of the study, a situation is cosidered, which requires monitoring of an industrial process manufacturing piston rings for an engine type. A piston ring is a metallic split ring attached to the outer diameter of a piston in an internal combustion engine or steam engine (Figure 11). The piston rings perform different functions, for example sealing the combustion chamber, improving heat transfer, maintaining the proper quantity in engine etc. Piston rings for an automotive engine are produced by a forging process.

Figure 11: Stainless Steel Piston Rings

It is required to control the inside diameters of the manufactured rings. The data consist of 40 samples of 5 piston rings and the inner diameter is measured to the nearest 100th of a millimeter (mm). The complete data are given in (Montgomery, 2020). For the current example, the initial 50 observations of the data are considered. The first 30 are used to estimate the unknown process parameters (phase I samples): $\hat{\mu} = 74.003$ and $\hat{\sigma} = 0.0116$. Whereas the next 20 are utilized as phase II samples to examine the performance of the three control charts: CUSUM, CCUSUM and DCUSUM, in the presence of the measurement errors. These 20 observations are altered, by adding 0.015 to each observation, in order to adjust the data for a shift. Three situations for measurement errors are taken: no errors ($\gamma = 0$), error variation is 50% of the process variation ($\gamma = 0.5$) and error variation is equal to the process variation ($\gamma = 1$). The control chart parameters for CUSUM, CCUSUM and DCUSUM are respectively: ($k = 0.5$, $h = 4.096$), ($k = 0.5$, $h = 4.101$) and ($k_1 = 0.32$, $k_2 = 0.78$, $h_1 = 7.04$, $h_2 = 2.89$). These parameters ensure an average run length of 370, when the process is working in-control. The control charts are shown in figures 12 to 14.

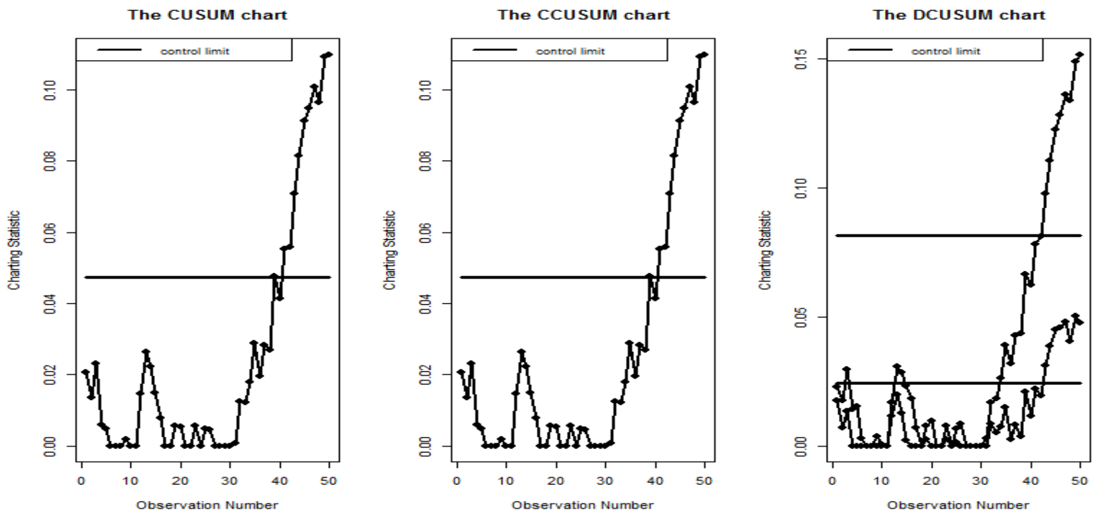


Figure 12: Control charts for CUSUM, CCUSUM and DCUSUM charts at $\gamma = 0$

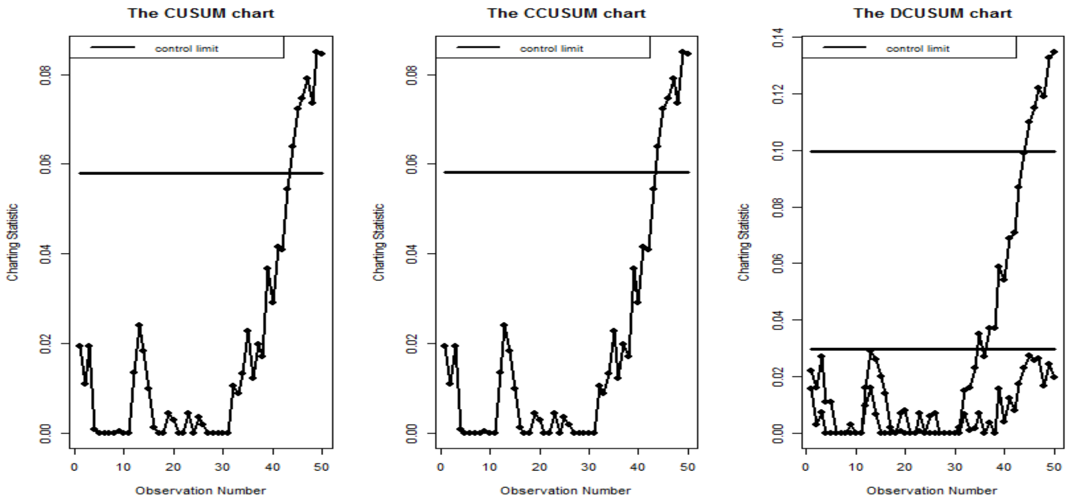


Figure 13: Control charts for CUSUM, CCUSUM and DCUSUM charts at $\gamma = 0.5$

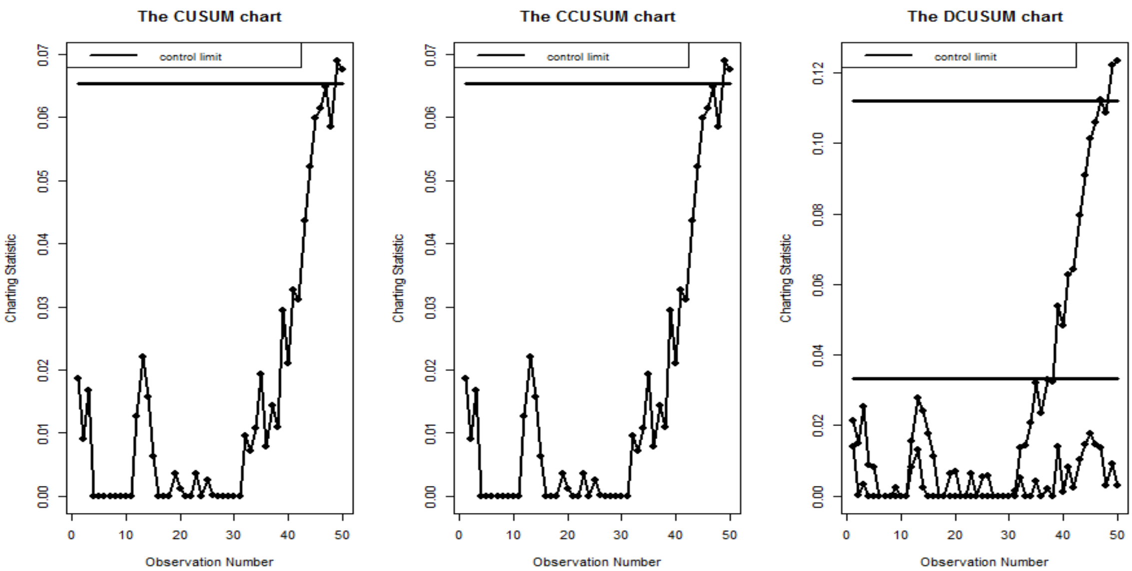


Figure 14: Control charts for CUSUM, CCUSUM and DCUSUM charts at $\gamma = 1$

The above figures vividly explain the deteriorating powers of the three charts, with the gradual intensity of measurement errors. When there is no error variation at work, the CUSUM and CCUSUM initiate a signal at sample number 39. With 50% variation, it becomes 44, while 100% variation aggravates it further to 49. Almost the similar pattern, the DCUSUM follows: with no errors it signals first at sample number 43 and with 50% and 100% error variations, these become respectively, 45 and 47.

6 Conclusion and Recommendations

The paper attempted to highlight the adverse effect of measurement errors on the performance of two widely recommended memory type control charts. The paper underscored a diminishing power of the charts to detect shifts, in the process parameters, in the presence of these errors. Although, the performance did get affected, by the measurement variation, but the general preference order of the charts remained unchanged. To counter such an effect, at first place, the measurements must be as precisely recorded as possible, and then repeated measurements could be used to reduce the effect. Note that the study is done by assuming that the process parameters are known. The study should be amended accordingly in case the assumption does not hold.

The study can be carried on further in a number of ways: more sophisticated models for the errors replicating different real-life situations, obtaining first an estimate of the error variance and then setting the control chart parameters, and studying the same effect on various other charts.

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